# Unsupervised Deep nets

Bryan Pardo
Northwestern University
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# Hebbian Learning

# Donald Hebb

- A cognitive psychologist active mid 20$^{th}$ century
- Influential book: The Organization of Behavior (1949)
- Hebb's postulate

"Let us assume that the persistence or repetition of a reverberatory activity (or "trace") tends to induce lasting cellular changes that add to its stability.… When an axon of cell *A* is near enough to excite a cell *B* and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that *A*'s efficiency, as one of the cells firing *B*, is increased.

# Pithy version of Hebb's postulate

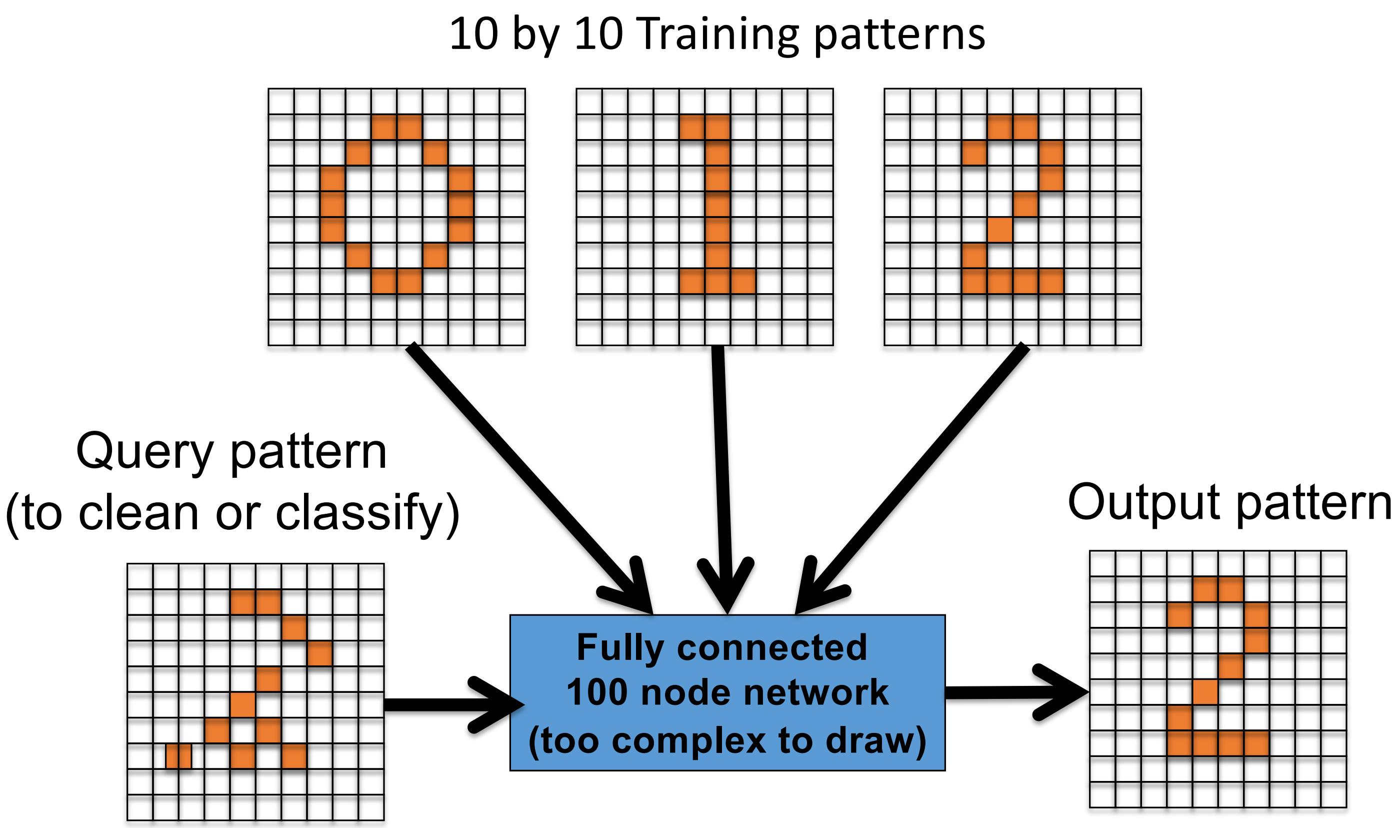
Cells that fire together, wire together.

# Hopfield networks

# Hopfield nets are

- “Hebbian” in their learning approach
- Old technology. These are proof of concept things from the 80s and earlier.
- Fast to train, slow to use
- Weights are symmetric
- All nodes are input & output nodes
- Use binary (1, -1) inputs and output
- Used as  Associative memory (image cleanup)Classifier
- Precursors to Auto Encoders and Restricted Boltzman Machines

# Using a Hopfield Net



# Training a Hopfield Net

- Assign connection weights as follows

$$w_{ij} = \begin{cases} \sum_{c=1}^{C} x_{c,i} x_{c,j} & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

$c$ index number for $C$ many class exemplars

$i, j$ index numbers for nodes

$w_{i,j}$ connection weight from node i to node j

$x_{c,i} \in \{+1, -1\}$ element i of the exemplar for class c

# Using a Hopfield Net

Force output to match an unknown input pattern

$$s_i(0) = x_i \qquad \forall i$$

Here, $s_i(t)$ is the state of node $i$ at time $t$ and $x_i$ is the value of the input pattern at node $i$

Iterate the following function until convergence

$$s_i(t+1) = \begin{cases} 1 & \text{if } 0 \leq \sum_{j=1}^{N} w_{ij} s_j(t) \\ -1 & else \end{cases}$$

Note: this means you have to pick an order for updating nodes.
People often update all the nodes in random order

# Using a Hopfield Net

Once it has converged…

FOR INPUT CLEANUP: You’re done. Look at the final state of the network.

FOR CLASSIFICATION: Compare the final state of the network to each of your input examples. Classify it as the one it matches best.

# Input Training Examples

Why isn't 5 in the set of examples?

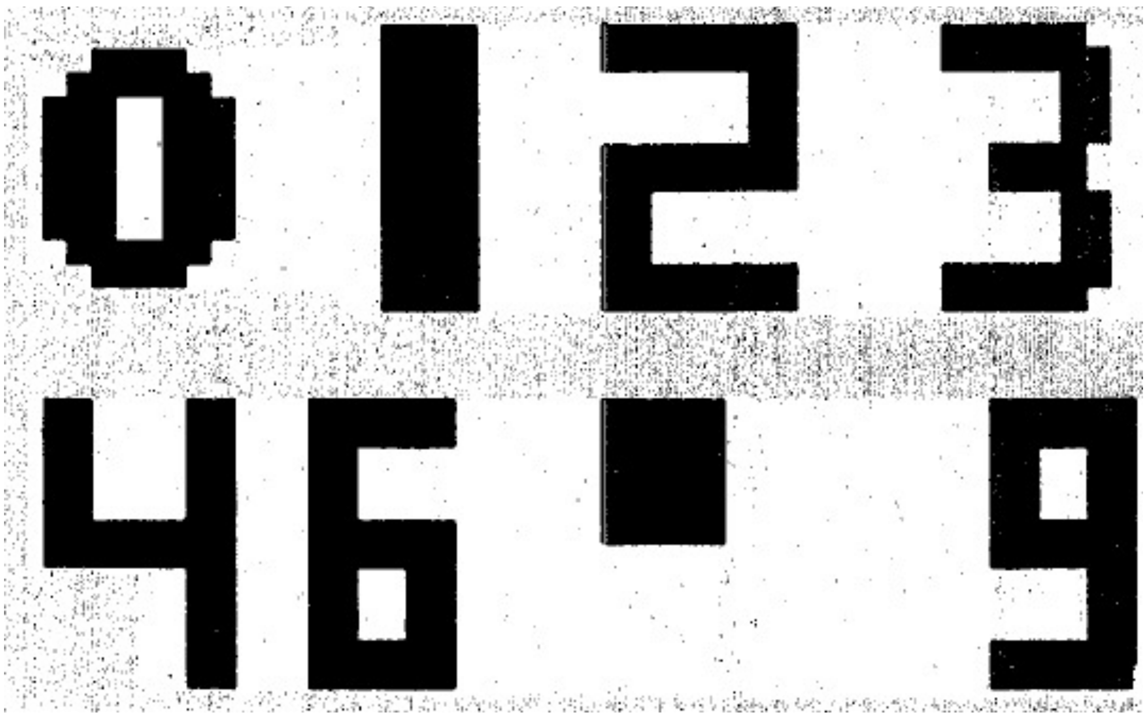
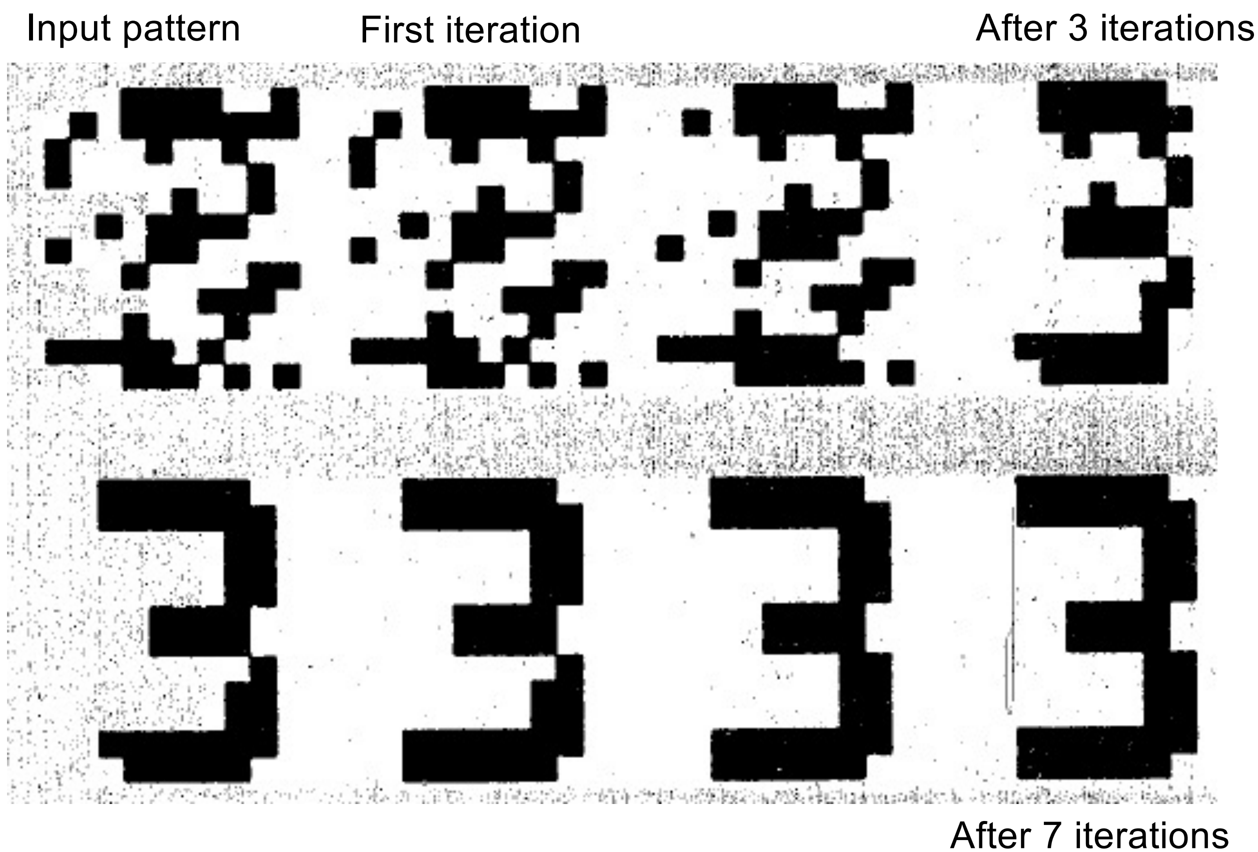


Image from:R. Lippman, An Introduction to Computing with Neural Nets, IEEE ASSP Magazine, April 1987

# Output of network over 8 iterations

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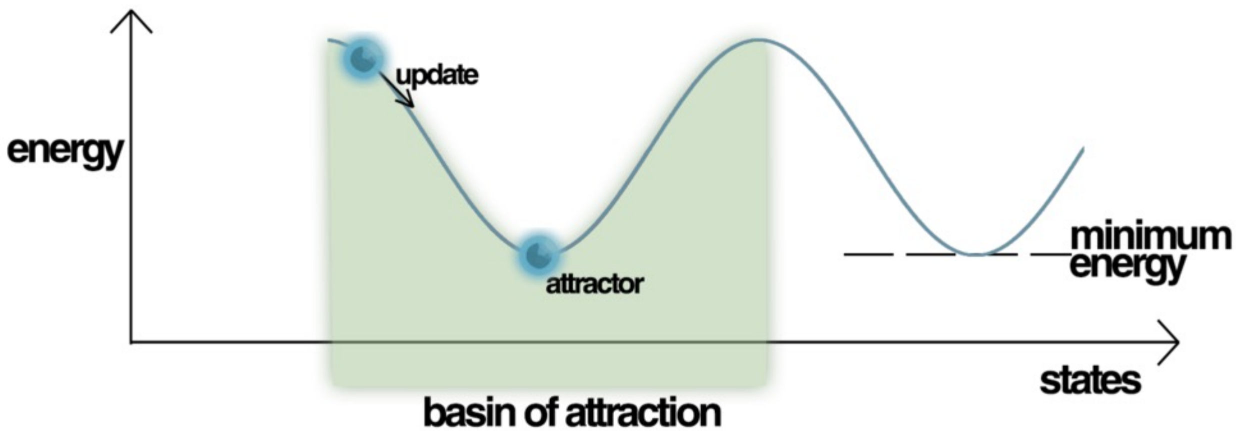
# Characterizing "Energy"




- As $s_i(t)$ is updated, the state of the system converges on an "attractor", where $s_i(t+1) = s_i(t)$
- Convergence is measured with this "Energy" function:

$$E(t) = -\frac{1}{2}\sum_{i,j} w_{ij} s_i(t) s_j(t)$$

Note: people often add a "bias" term to this function. I'm assuming we've added an extra "always on" node to make our "bias"

Image from: http://en.wikipedia.org/wiki/Hopfield_network#mediaviewer/File:Energy_landscape.png

# Limits of Hopfield Networks

- Input patterns become confused if they overlap
- The number of patterns it can store is about 0.15 times the number of nodes
- Retrieval can be slow, if there are a lot of nodes (it can take thousands of updates to converge)

# Restricted boltzman machine (RBM)

# About RBNs

- Related to Hopfield nets

- Used extensively in Deep Belief Networks

- You can’t understand DBNs without understanding these

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- Related to Hopfield nets
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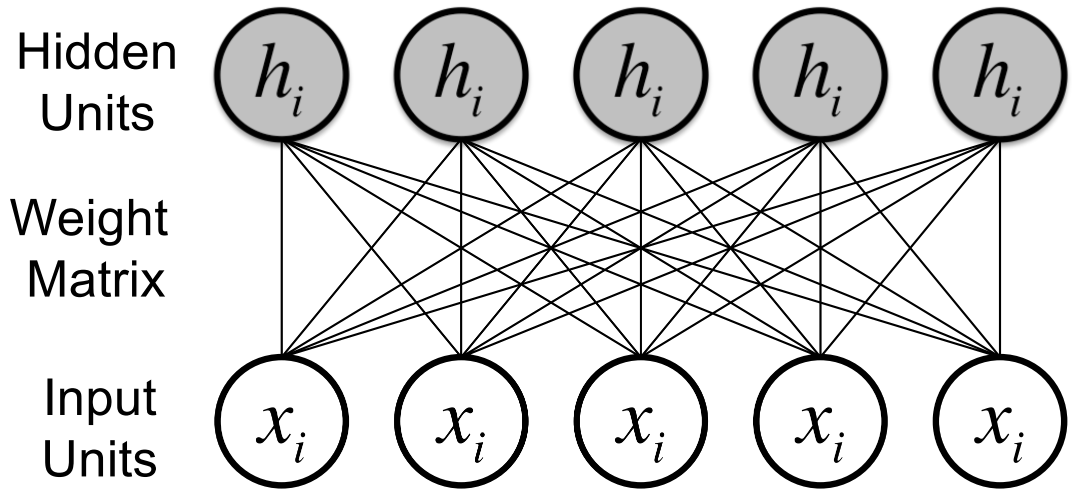
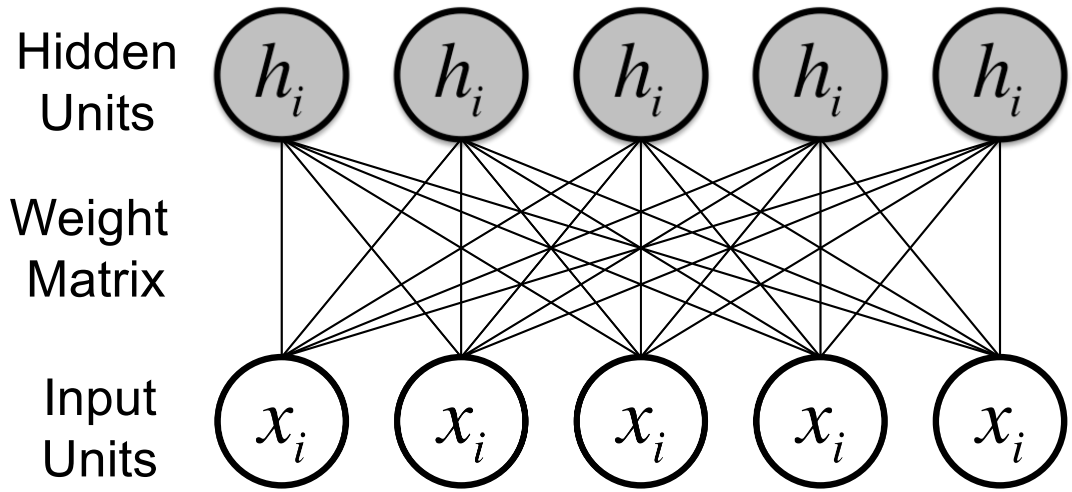
# Standard RBM Architecture




2 layers (hidden & input) of Boolean nodes

Nodes only connected to the other layer

# Standard RBM Architecture




Setting the hidden nodes to a vector of values updates the visible nodes…and vice versa

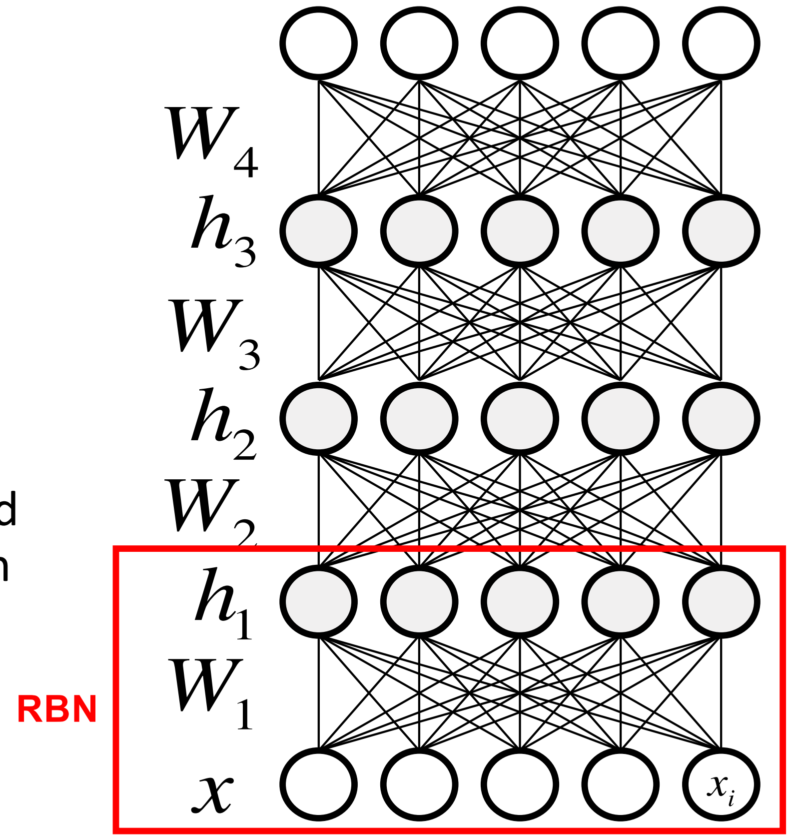
# Contrastive Divergence Training

1. Pick a training example.
2. Set the input nodes to the values given by the example.
3. See what activations this gives the hidden nodes.
4. Set the hidden nodes at the values from step 3.
5. Set the input node values, given the hidden nodes
6. Compare the input node values from step 5 to the the input node values from step 2
7. Update the connection weights to decrease the difference found in step 6.
8. If that difference falls below some epsilon, quit. Else, go to step 1.

# Deep BELIEF Network (DBN)

# What is a Deep Belief Network?

- A stack of RBNS
- Trained bottom to top with Contrastive Divergence
- Trained AGAIN with supervised training (similar to backprop in MLPs)



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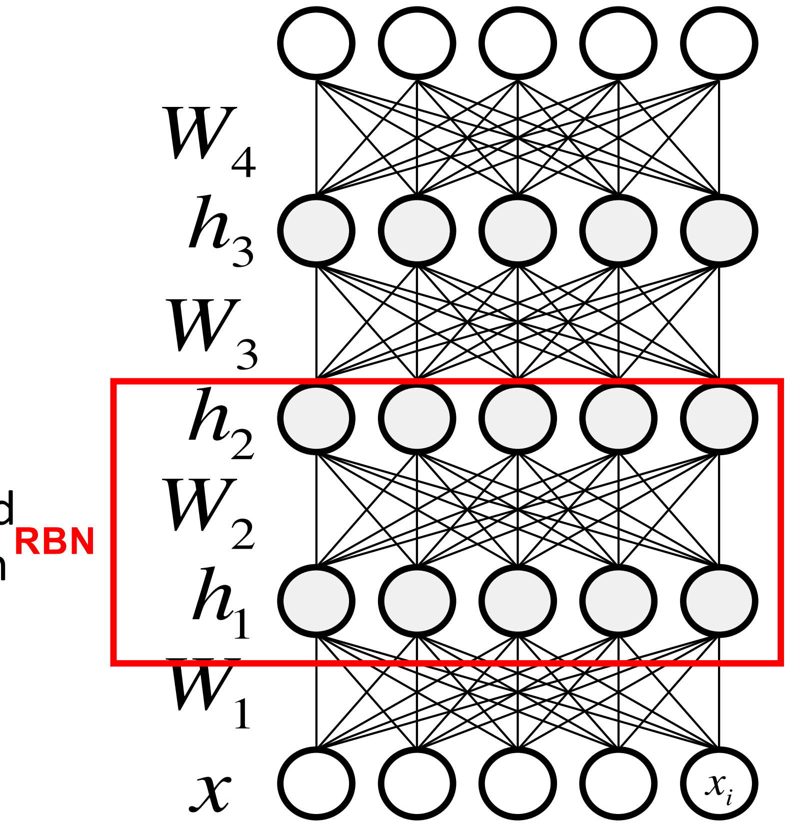

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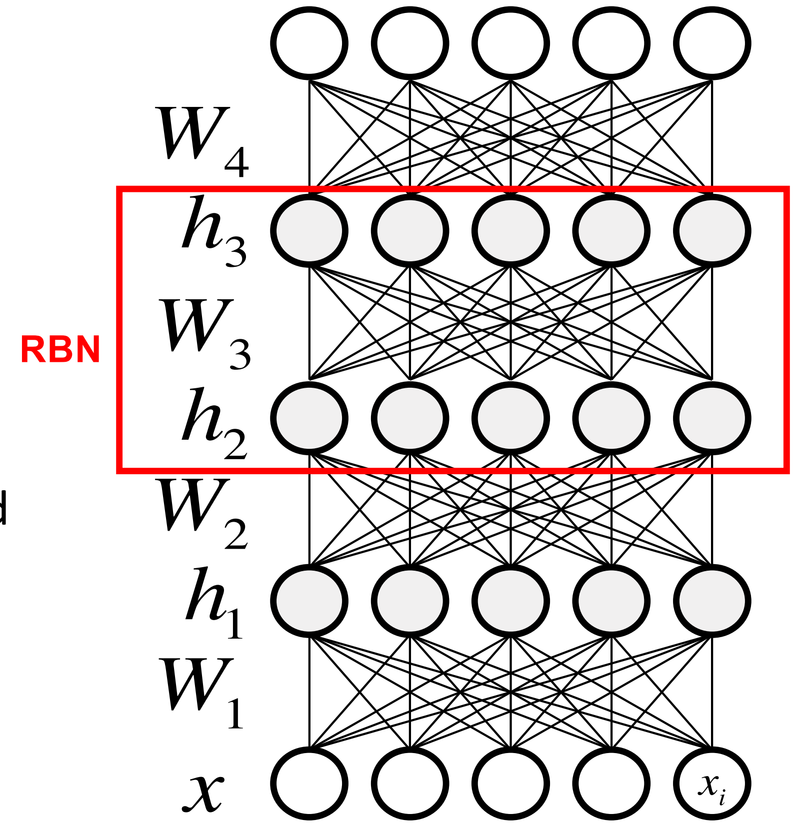

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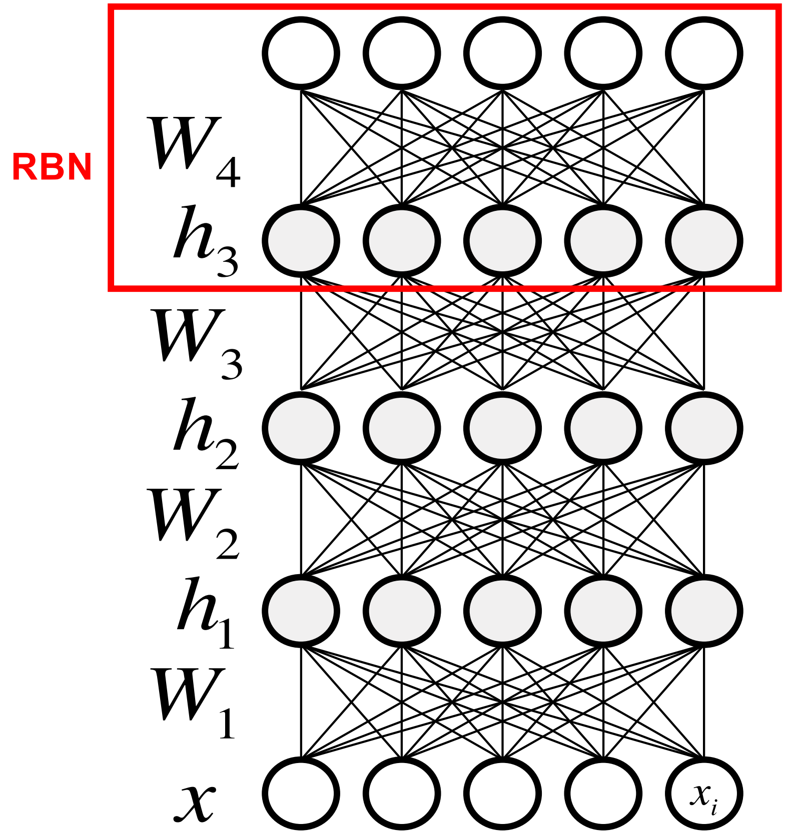


# Why are DBNs important?

- Around 2005 they were state-of-the-art systems for doing certain recognition tasks
  - Handwritten digits
  - Phonemes

- They got the whole “Deep learning” thing going. Before these, nets maxed out at about 4 layers.

- They had a good marketing campaign “Deep learning” vs “shallow learning”

# Why not use standard MLP training?

- Fading signal from backprop. (this was later solved with normalization layers)

- The more complex the network, the more likely there are local minima

- Memorization issues

- Training set size and time to learn

# Benefits

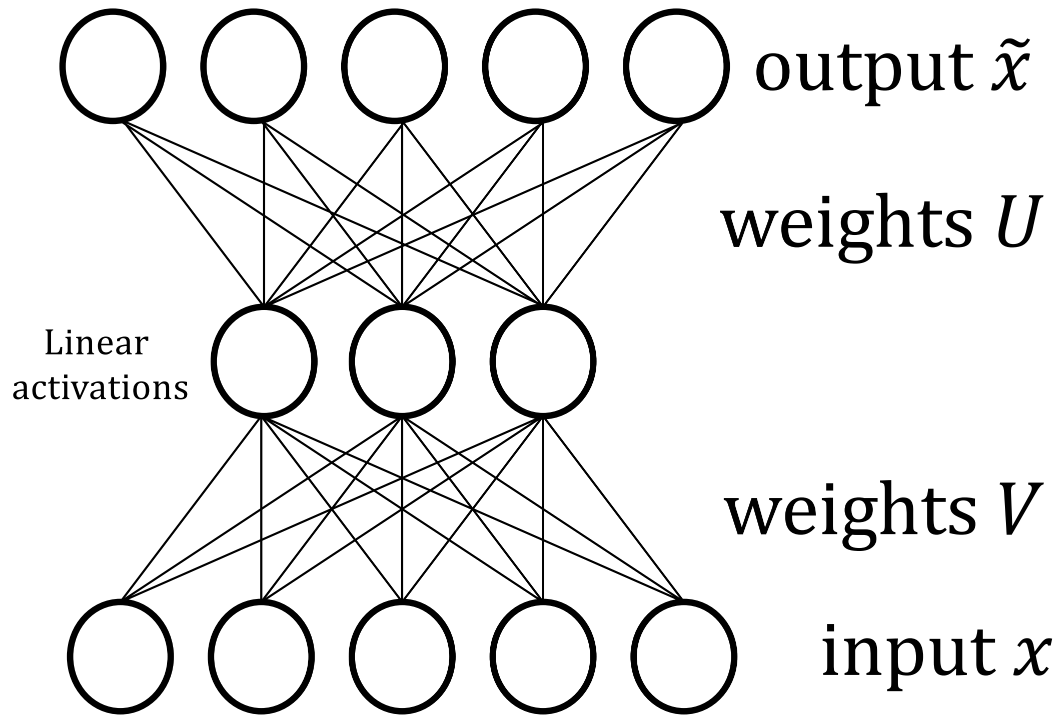
- Allowed relatively deep networks (e.g. 10 layers), compared to regular multilayer perceptrons with sigmoid activations

- In the mid 2000's this approach made networks better than anything else out there for some problems (e.g. digit recognition, phoneme recognition)

- Today, they've been superseded by other approaches

# Autoencoders

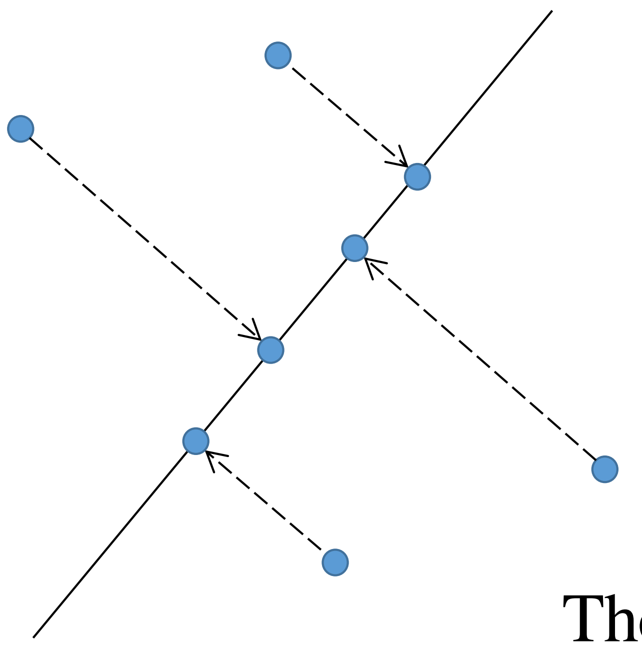
# Simplest Autoencoder: Linear model, 1 hidden layer




$$\text{loss } \mathcal{L} = \|x - \tilde{x}\|^2$$
$$\tilde{x} = UVx$$

Maps $d$ dimensional input $x$ to $k$ dimensional embedding subspace $S$

# Simplest Autoencoder: Linear model, 1 hidden layer



$$\text{loss } \mathcal{L} = ||x - \tilde{x}||^2$$

$$\tilde{x} = UVx$$

Maps $d$ dimensional input $x$ to $k$ dimensional embedding subspace $S$

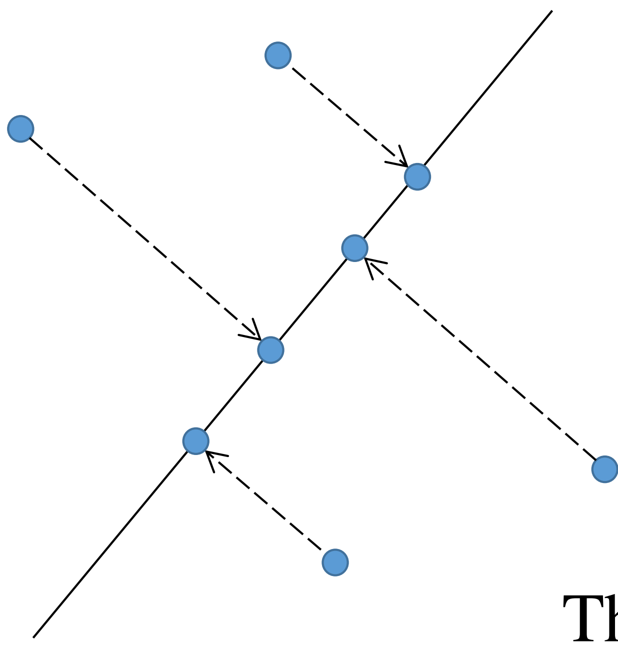
The projection of $x$ onto $S$ is the point in $S$ which minimizes the 2-norm distance to $x$.

# Simplest Autoencoder: Principal Component Analysis

The linear autoencoder learns $U = Q$ and $V = Q^{T,}$ where $Q$ is an orthonormal basis for $S$.

$$\text{loss } \mathcal{L} = \|x - \tilde{x}\|^2$$

$$\tilde{x} = UVx$$



Maps $d$ dimensional input $x$ to $k$ dimensional embedding subspace $S$

The projection of $x$ onto $S$ is the point in $S$ which minimizes the 2-norm distance to $x$.

# This was just to get the idea…

- You wouldn’t actually do this by training a neural net.

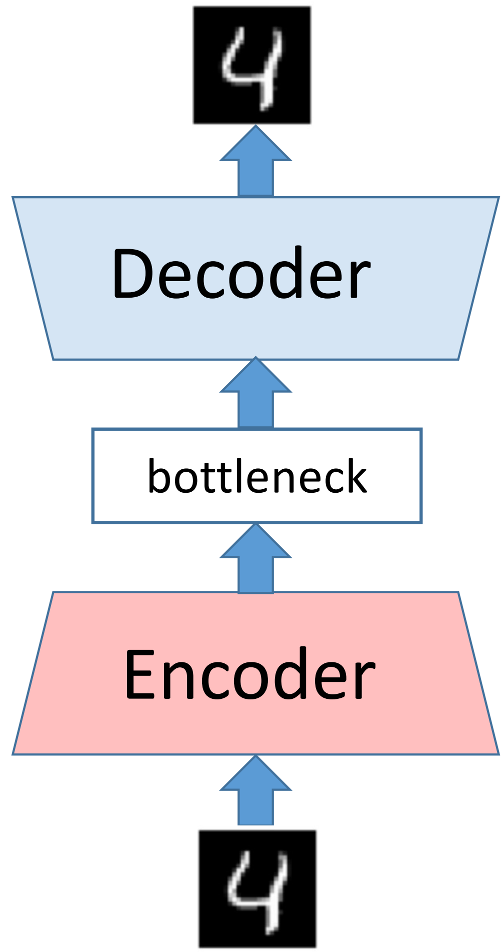
- The standard algorithm is called principal component analysis (PCA).

- Read about it here:

https://en.wikipedia.org/wiki/Principal_component_analysis

# More generally

- With multiple layers & nonlinear activations we can map on to a nonlinear embedding space
- We can represent complex data this way and use the encoder as the input to a supervised network
- This lets us learn features from unlabeled data, which is far easier to get than labeled data.



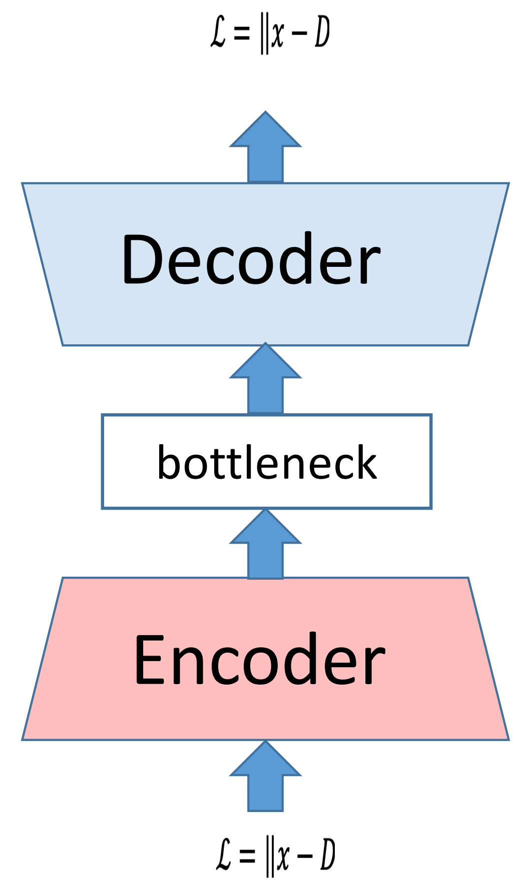
# Sparsity constraints are good

- Often better data representations can be gotten by adding a sparsity constraint to the loss function.

$$\mathcal{L} = ||x - D(E(x))||^2 + ||W||^2$$

Here W is the network weights.

- This is a sparse autoencoder.

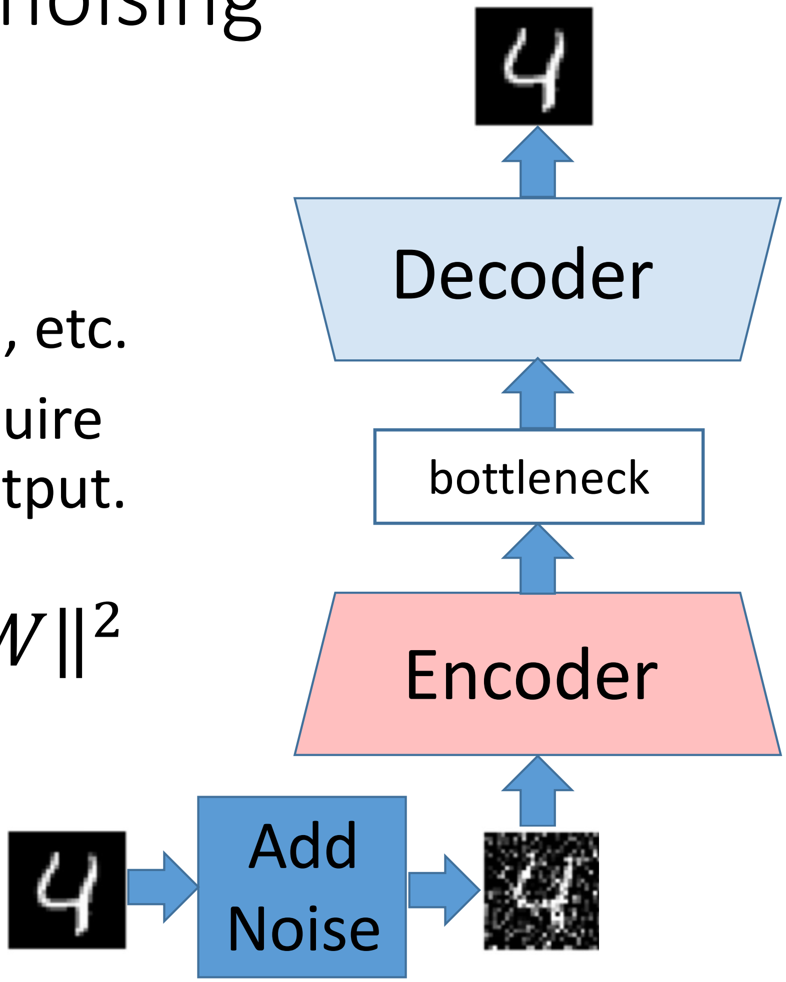


# They're also great for denoising

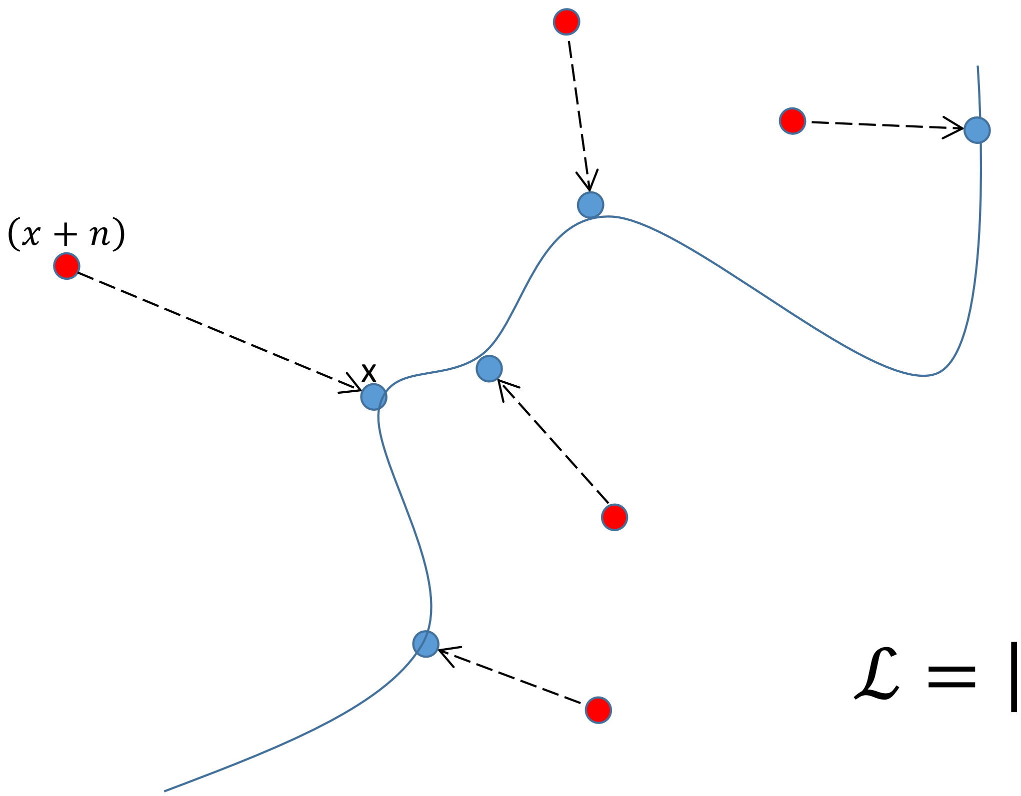
- Autoencoders can be trained to remove noise from images, speech, etc.
- Just add noise to the input and require reconstruction of the non-noisy output.

$$\mathcal{L} = ||x - D(E(x + n))||^2 + ||W||^2$$

- This is a denoising autoencoder.



# DAE: Maps to a learned non-linear embedding

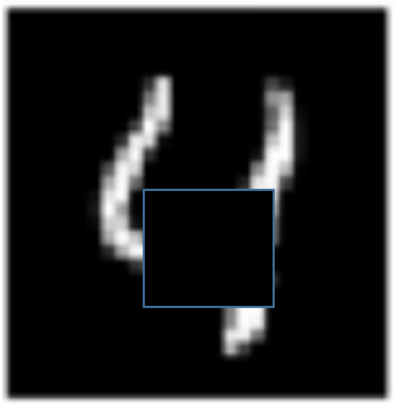



Same idea as PCA, in some sense, but nonlinear….

$$\mathcal{L} = \|x - D(E(x+n))\|^2 + \|W\|^2$$

# Also great for imputation/inpainting

- If the “noise” we add is masking out large patches….



- We can train it to fill in blanks.