

Topic

Filters, Reverberation & Convolution

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Lecture Outline

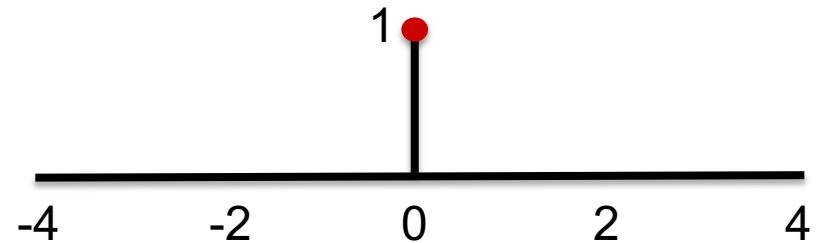
- What's an Impulse?
- LTI systems
- Frequency selective filters
- Delay and moving average
- Reverberation

The impulse function

Impulse response

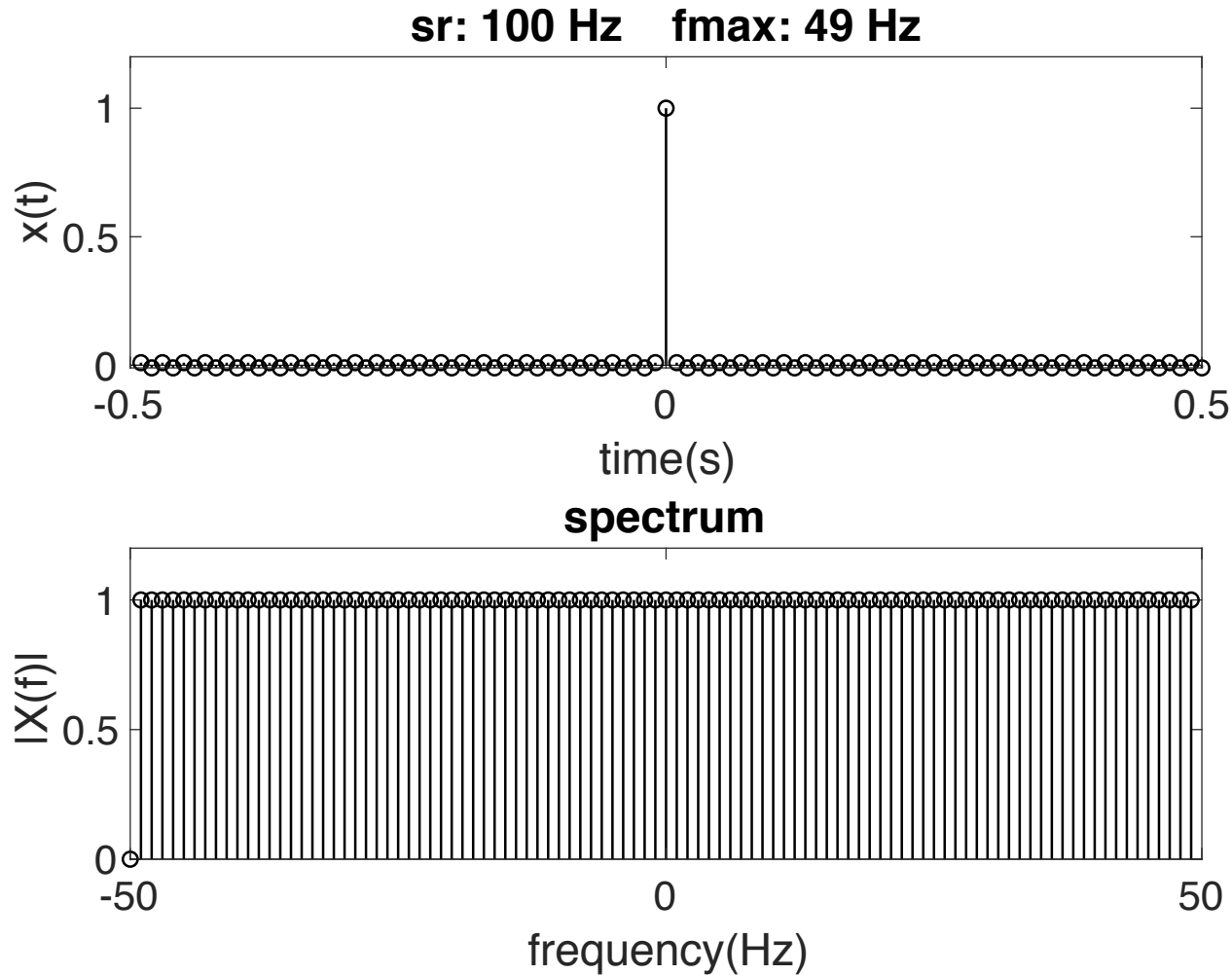
- An “impulse” is this signal:

$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{else} \end{cases}$$



- The next slide shows an impulse (top) and the frequency representation of that impulse you get by plotting the absolute value of the Fourier transform of that impulse.
- Note, the impulse shows energy at ALL frequencies of analysis.

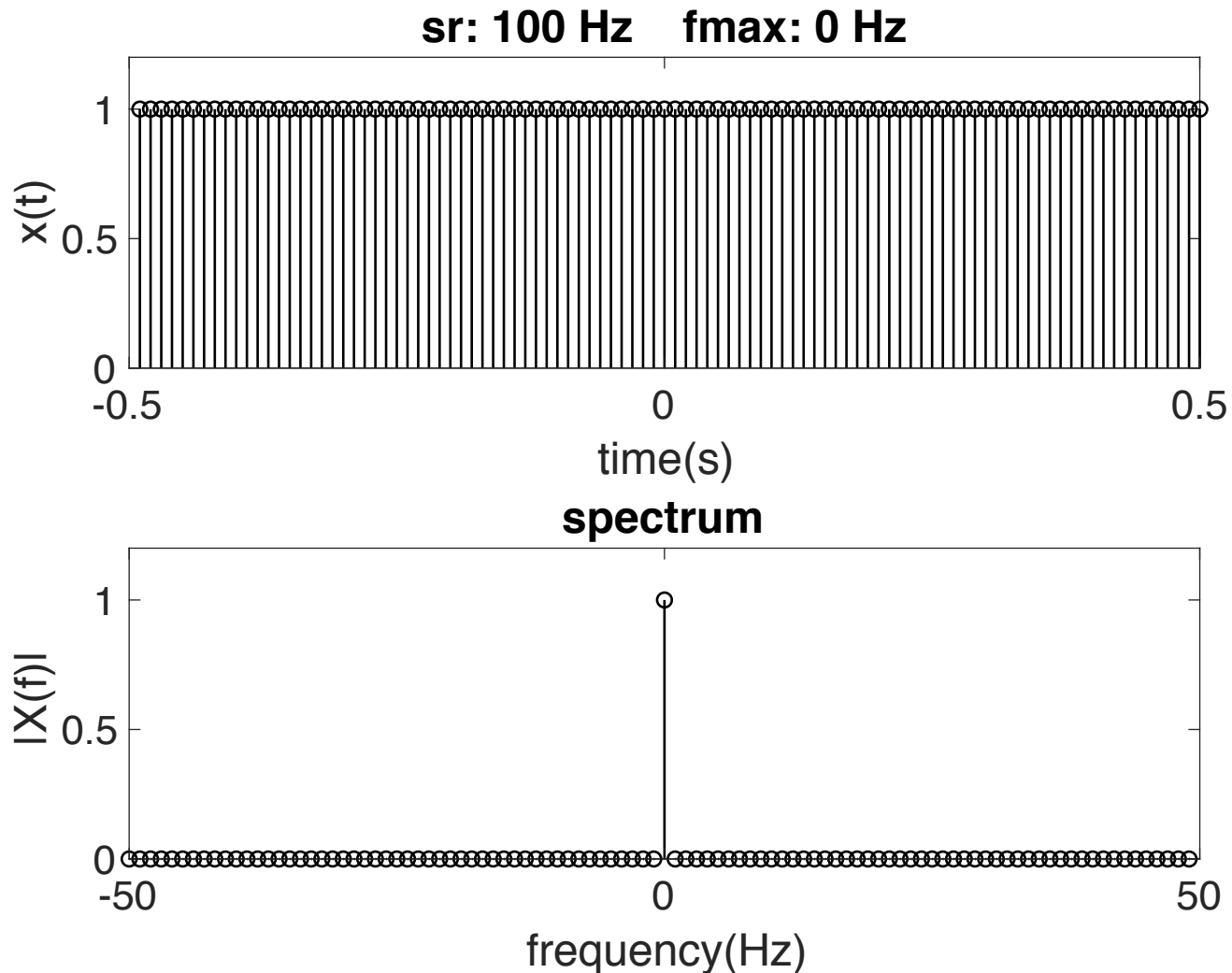
Impulse function



Let's look at the D.C offset

- Next slide, you'll see a time-domain signal (top panel) that is all 1. This is a constant signal.
- When you take the FFT and display the magnitude spectrum (bottom panel), you get just one non-zero value, at the 0 frequency. This is the D.C. offset.
- The DC offset is basically the opposite of the impulse function from the previous slide: all times are non-zero, but only one frequency is non-zero.

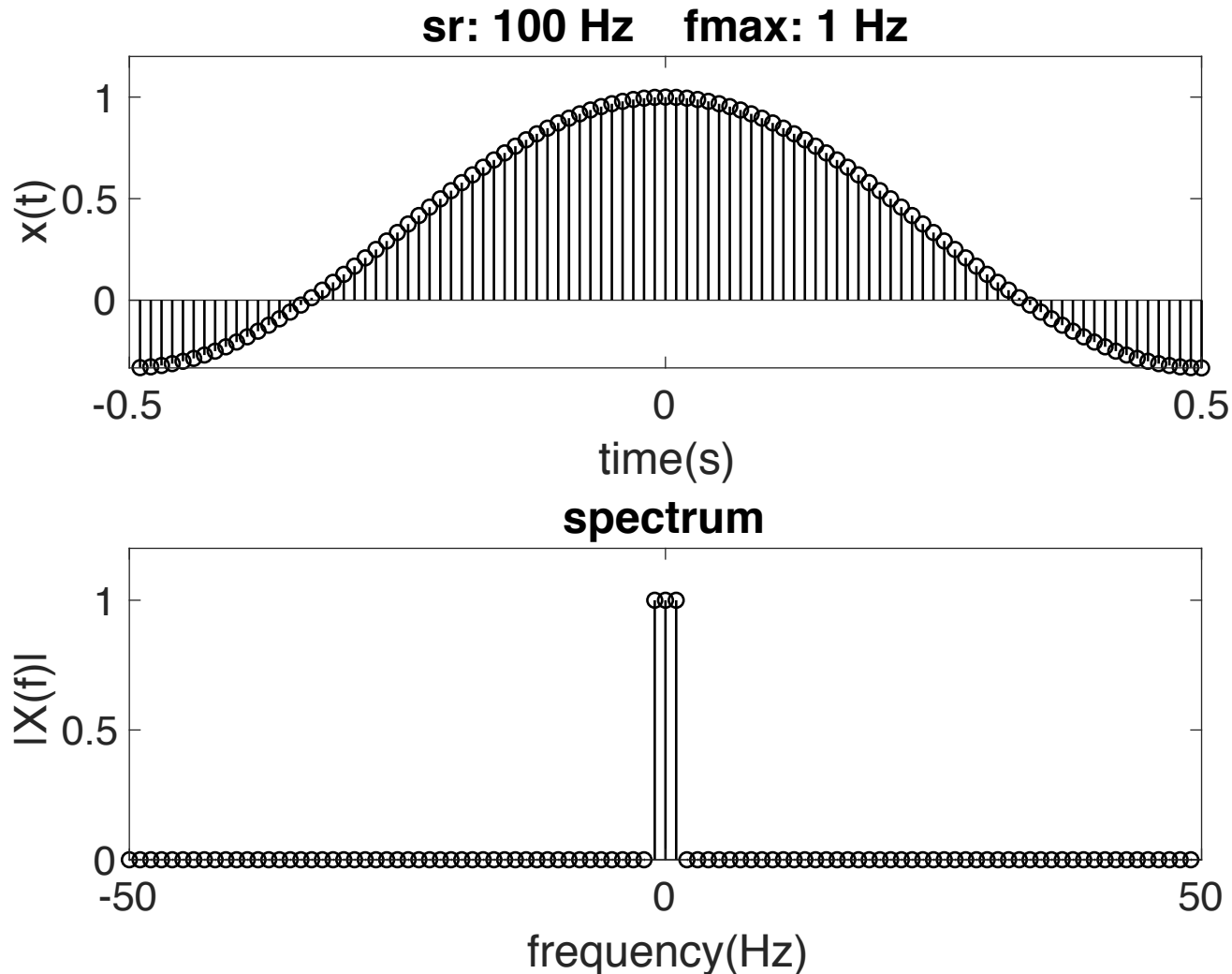
DC Offset in time and frequency



Making it more broadband

- Now, let's take our DC offset and make it a touch more broadband.
- We'll add energy in the frequency domain X by setting the 1st frequency of analysis (in both the positive and negative frequencies) to 1.
- Then, we do the inverse Fourier transform to see what the signal looks like.
- Note that, this changes what happens in the time domain. Now the energy starts to concentrate towards the middle.

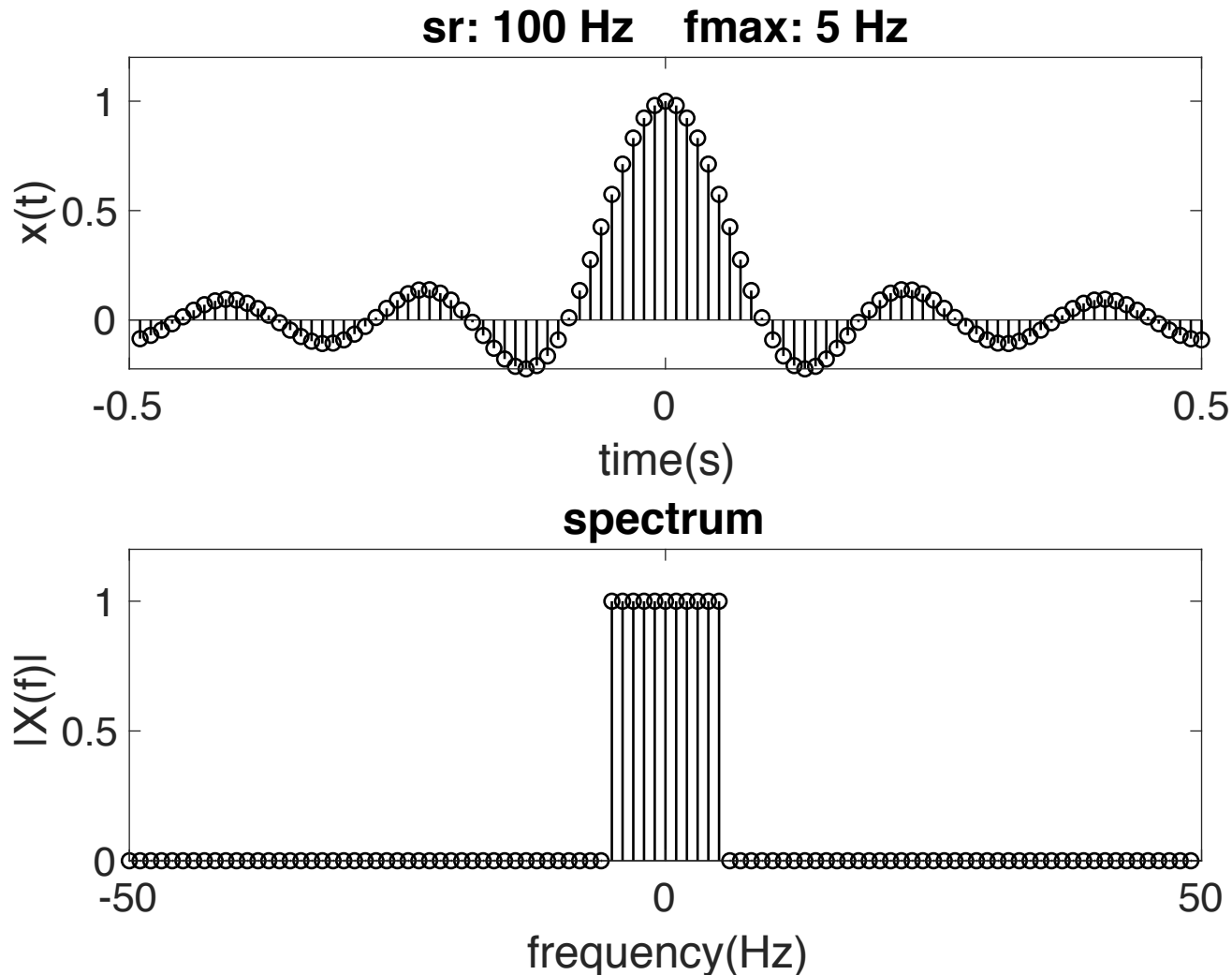
Adding energy at low frequency



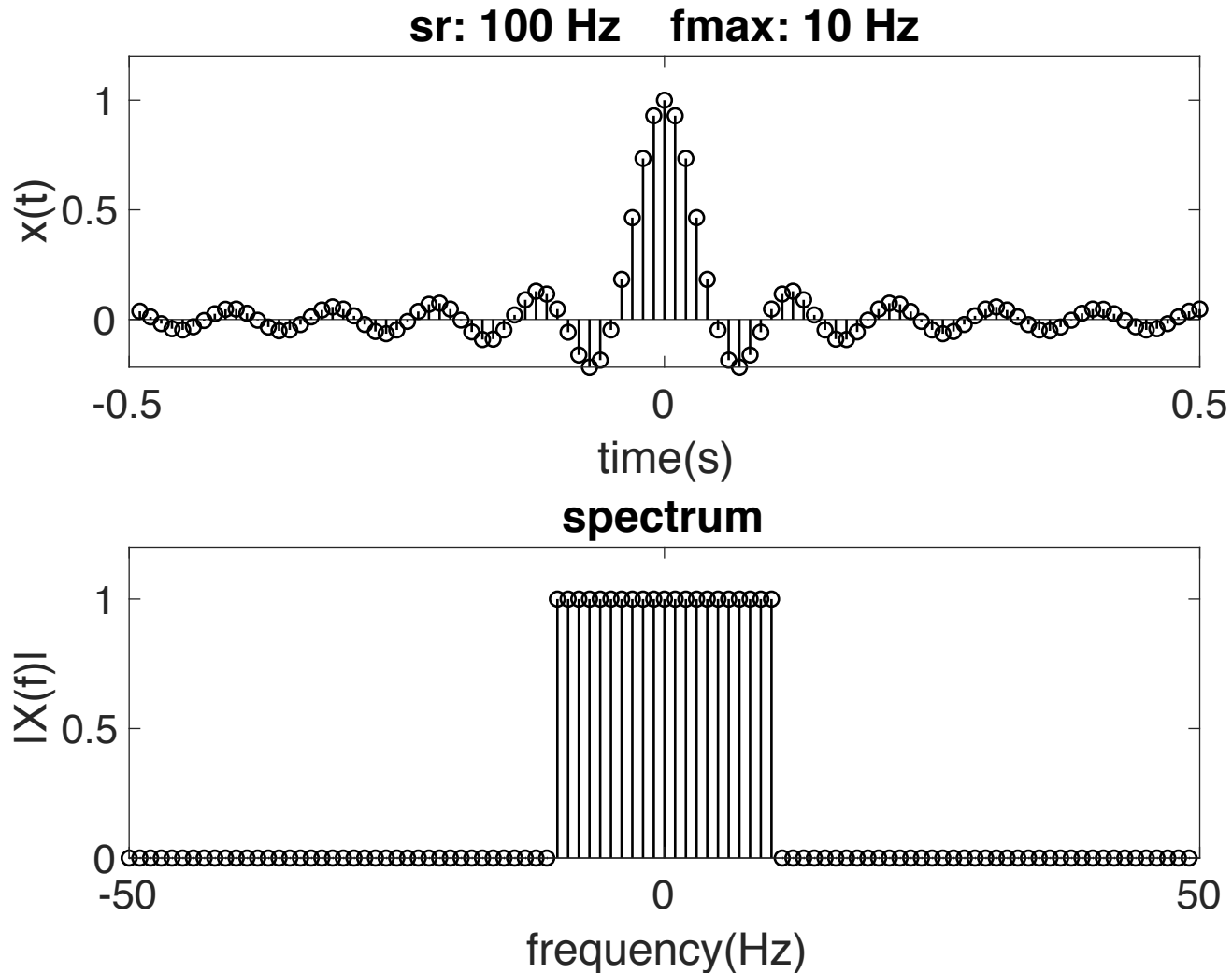
Rinse and repeat

- Over the next few slides, we'll keep adding energy at new frequencies and then take the inverse Fourier transform to see what happens in the time domain.
- As the frequency representation become more broadband (i.e. multiple frequencies have a lot of energy), the time representation of the signal becomes more and more centered on a single point in time.

Getting more broadband

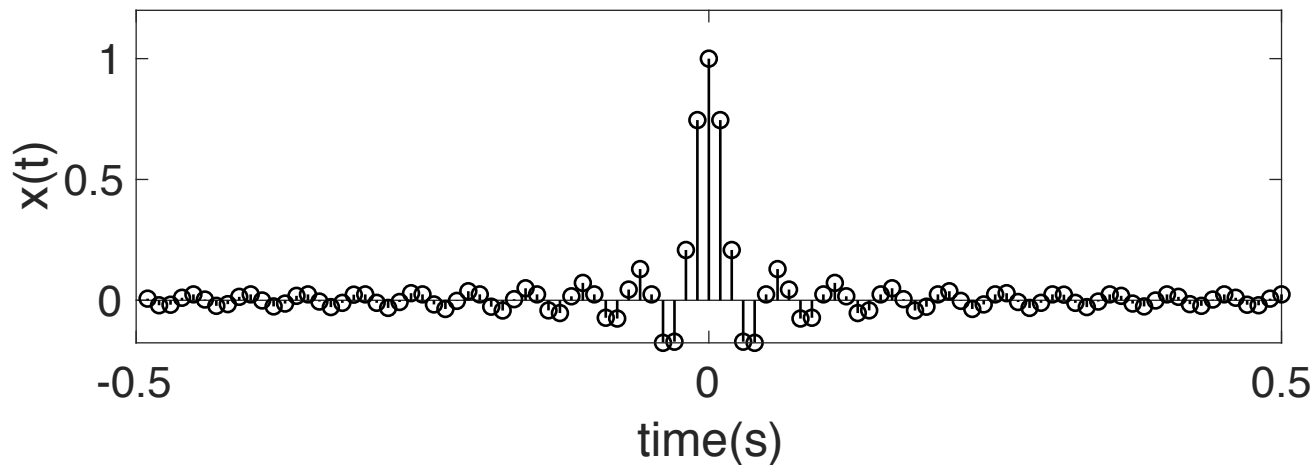


Even broader

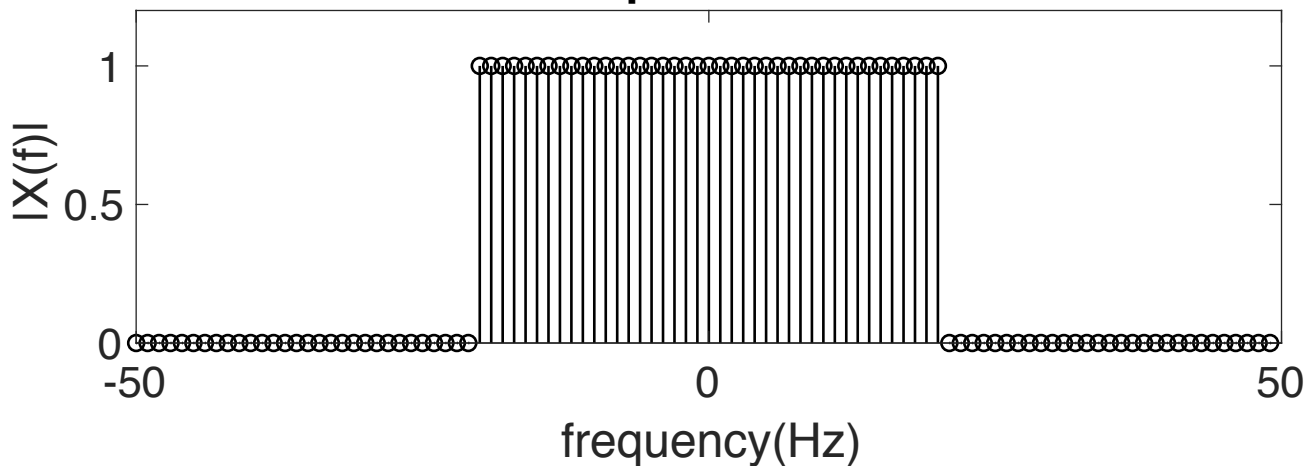


Broader still

sr: 100 Hz fmax: 20 Hz



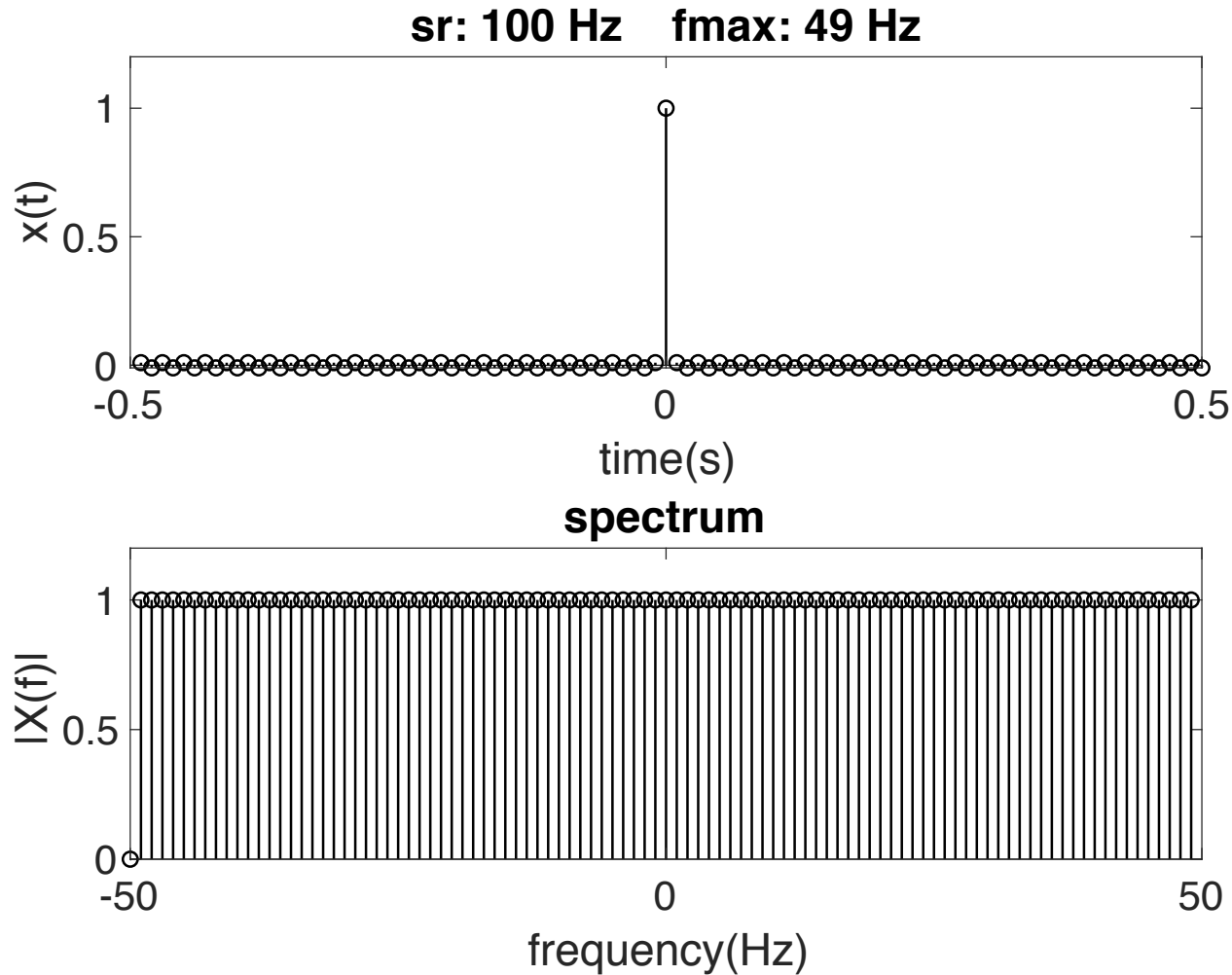
spectrum



How about a signal that has equal energy at all frequencies?

- Next slide, you'll see a signal that has equal energy at all frequencies.
- As you can see, when we do that, we're back to the impulse function.

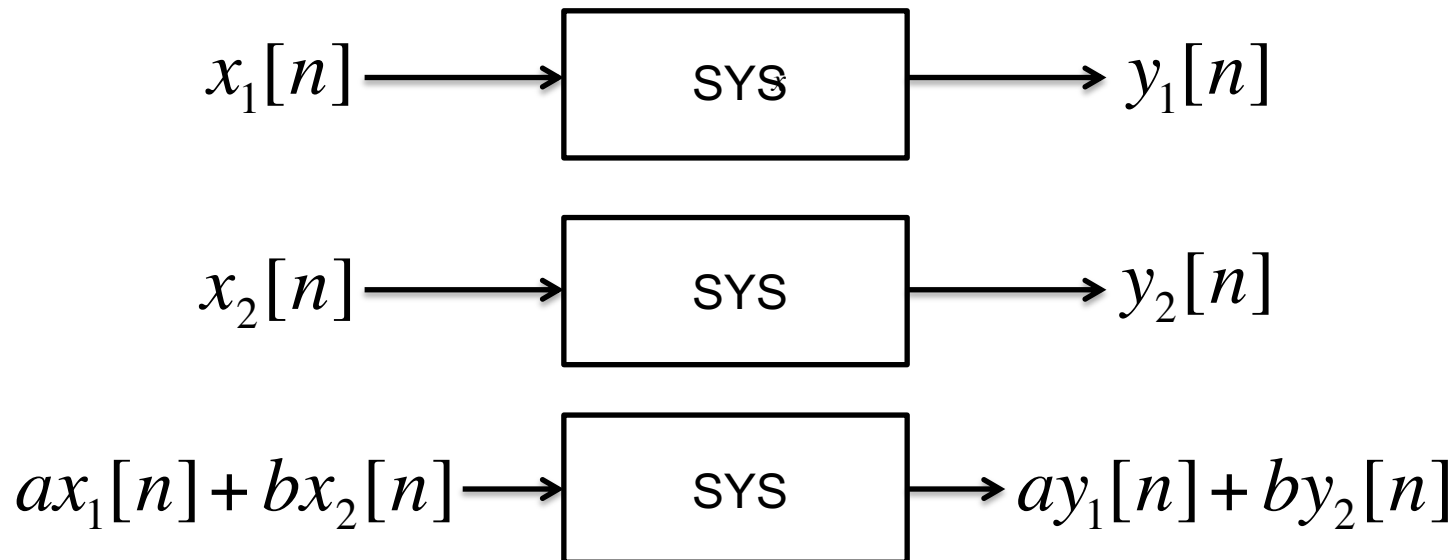
Impulse function



Linear, Time-invariant systems

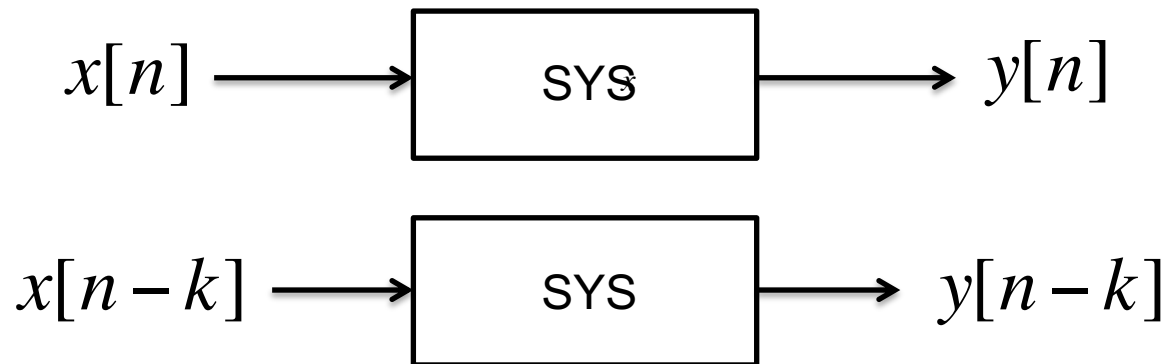
Linearity & Time Invariance

- A system is called linear if it satisfies the superposition property:



Linearity & Time Invariance

- A system is called time invariant if its behavior does not change over time:



LTI systems

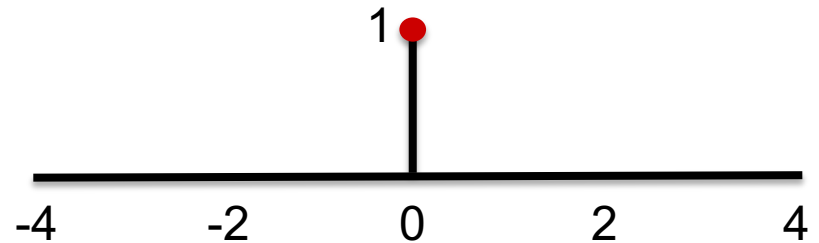
- Why do we like to think of systems we work with as LTI?
- Examples of linear systems?
- Examples of time invariant systems?

(We're going to assume rooms, like a classroom or a church, are LTI systems)

Impulse response

- Recall that an “impulse” is this signal:

$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{else} \end{cases}$$




- The ***impulse response*** $h[n]$ of a system is the output of the system when the input is an impulse.
- The ***frequency response*** $H(\omega)$ of a system is the Fourier transform of its impulse response $h[n]$.

Impulse response

- An LTI system is fully identified by its impulse response (or frequency response), because...
 - An arbitrary signal $x[n]$ is the sum of scaled and shifted impulse functions:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k]$$

- Then if we have $h[n]$, by assuming linearity and time invariance we can find the response to $x[n]$


$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n - k]$$


Does this formula look familiar?

Impulse response

- Look! We're back to convolution!

Definition of convolution

$$(x * h)[n] \equiv \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$


$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

Does this formula look familiar?

Frequency selective filters

- Frequency selective filters we use in this course are a subset of LTI system.
- The output of filters will be computed via **convolution in the time domain**, or equivalently, via **multiplication in the frequency domain**.

Filtering, practically speaking

- I can filter a signal $\mathbf{x}[n]$ with filter $\mathbf{h}[n]$ get a filtered signal $\mathbf{y}[n]$ by doing convolution between $\mathbf{x}[n]$ and $\mathbf{g}[n]$
- Time-domain convolution takes $O(n^2)$ time. Too long!
- The circular convolution theorem says this...

$$\mathbf{y} = \mathbf{h} * \mathbf{x} = \text{ifft}(\mathbf{H} \cdot \mathbf{X})$$

- Therefore, we take the fft of \mathbf{h} to get \mathbf{H} and the fft of \mathbf{x} to get \mathbf{X} .
- Then we element-wise multiply \mathbf{X} by \mathbf{H}
- Then we do the inverse fft on the result to get \mathbf{y}

One caveat

- Let's say both \mathbf{x} and \mathbf{h} are \mathbf{n} samples long.
- According to the definition of convolution, you should end up with something that is $2\mathbf{n}-1$ samples long (Go back and look at the python code for of convolution in the course slides)
- For this to happen in frequency, you have to zero-pad both \mathbf{x} and \mathbf{h} to be length $2\mathbf{n}-1$ when you do the Fourier transform.
- This is easy to do as a parameter you pass the fft function.

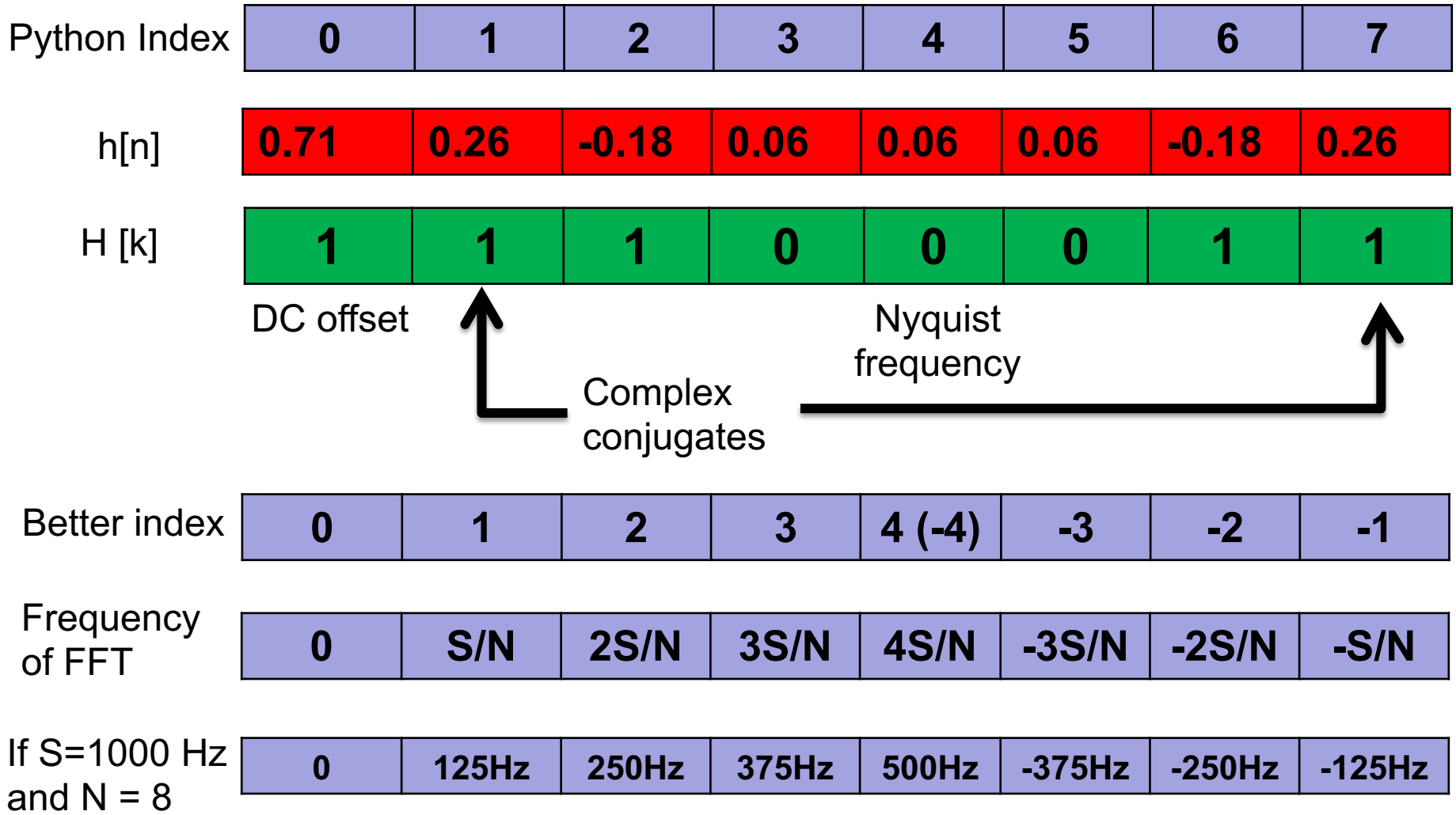
FIR Filter

- FIR means “Finite Impulse Response”
- Means it will stop making noise once you stop putting noise through it
- There are also Infinite Impulse Response (IIR) filters. These have feedback
- To find out more about IIR filters, take a DSP class.

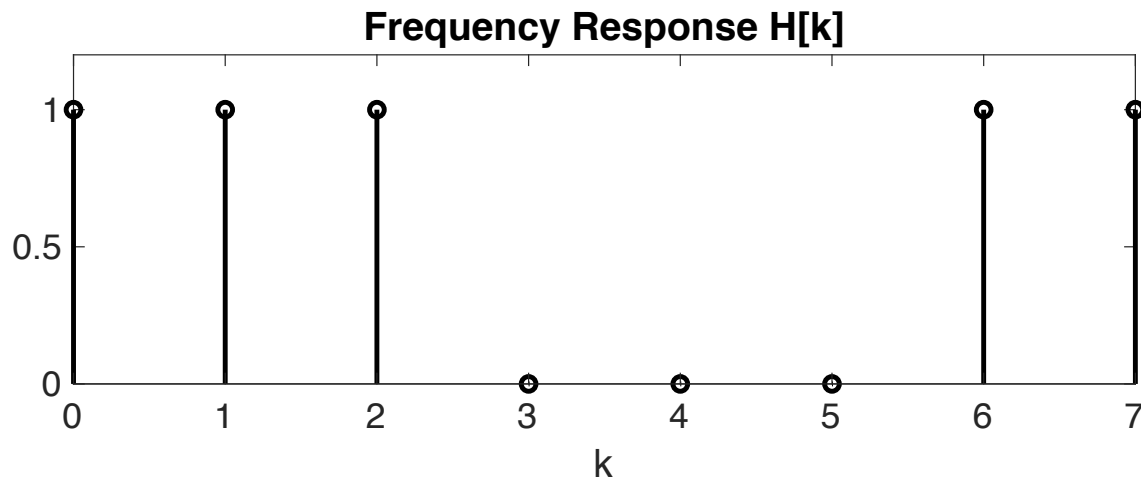
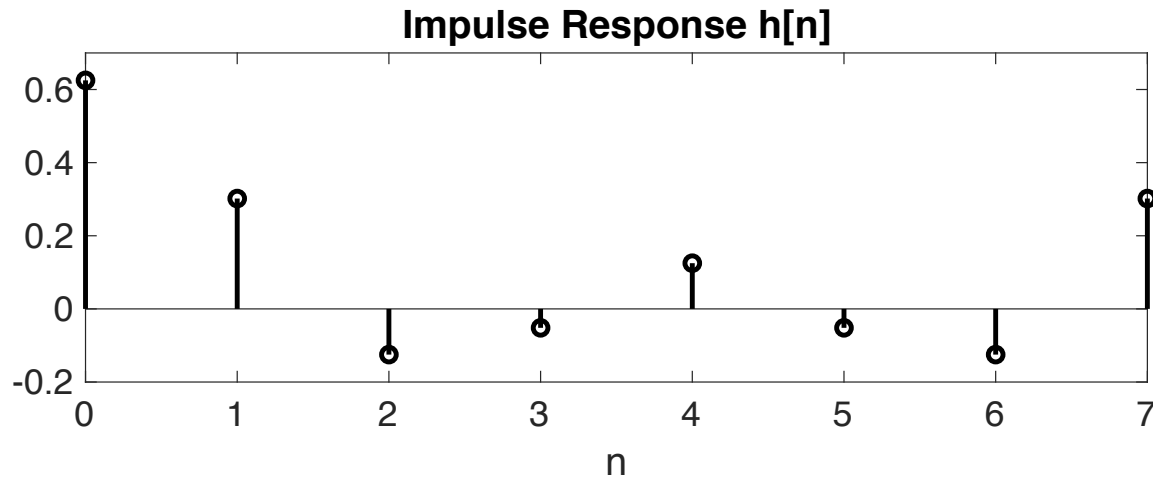
Building a low-pass FIR filter

- Pick width of your LOW pass band
(0 Hz to ω_c Hz, where ω_c can be up to the Nyquist rate)
- Create a desired frequency response (don't forget the mirror frequencies above the Nyquist rate).
- Take the IFFT of the frequency response
- This is your impulse response function.

An 8-point 250 Hz low-pass filter



The same 8-point low-pass filter (another view)

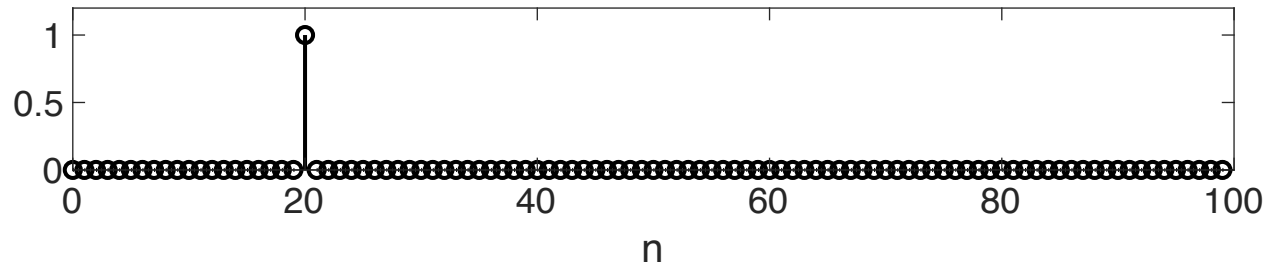


What is reverberation?

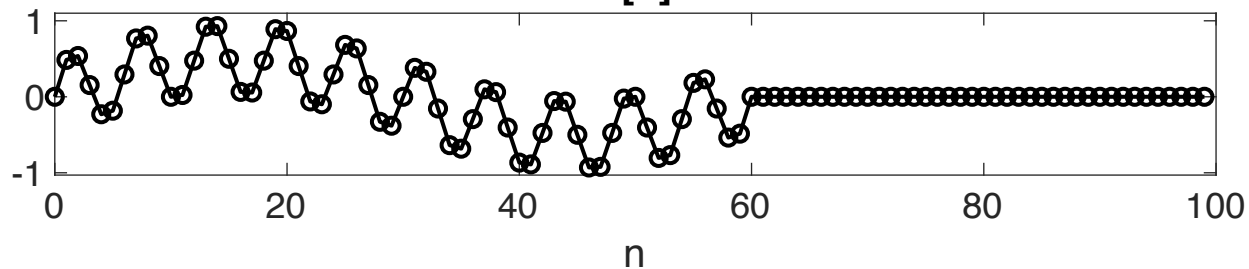
- Reverberation is made of echoes
- Echoes are delayed copies of the original sound
- In the physical world these are caused by reflections off of walls and other surfaces
- In the digital world, this is done with impulse response functions and convolution

Example: Delay Operator (aka a single echo)

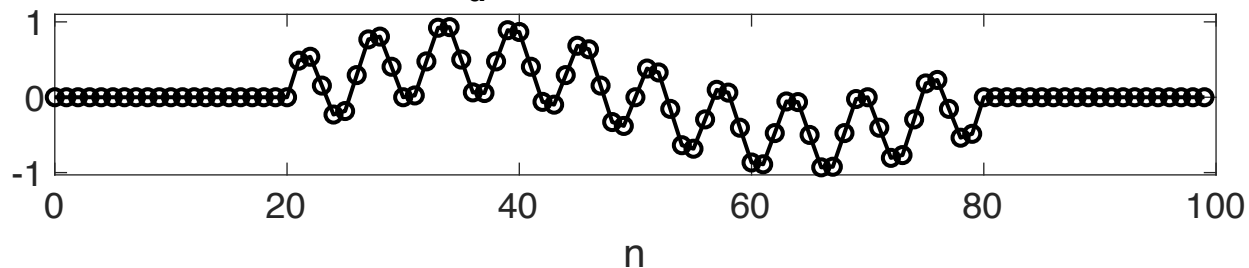
$$h_d[n] = \delta[n-20]$$



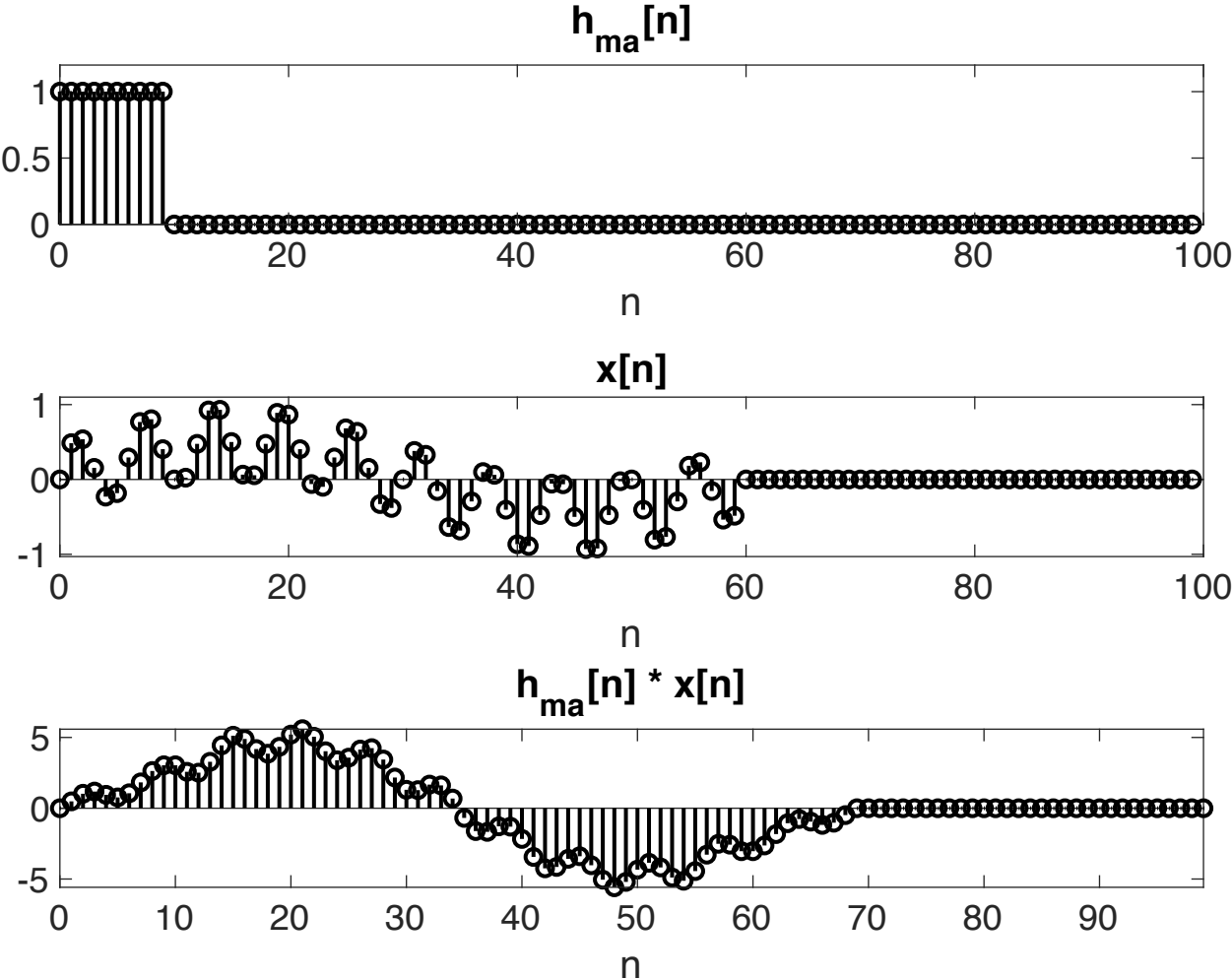
$$x[n]$$



$$h_d[n] * x[n] = x[n-20]$$



Several delayed copies = a moving average



Moving average acts as a low-pass filter

- The moving average operator makes copies of the signal and sums them.
- You're summing multiple, slightly-offset copies.
- This causes high-frequency events to average out in the wash (low-pass filtering).
- What would happen if the copies are offset by the exact period of some frequency in the audio?

A room is a filter



Carnegie Hall

Reverberant Rooms

- ...are louder (why?)
- ...make speech harder to understand (why?)
- ...may emphasize certain frequencies (why)?

They are all strongly related

- A room can be modeled as a **filter**
- **Reverb** is the effect of room/filter on sound signals
- Mathematically, adding reverb to sound signals (filtering) is performed via **convolution**

Making a convolution reverb

- The impulse response function of a room, \mathbf{h} , captures the echoes (reverberation) that happen when you make a sound in that room.
- You can use \mathbf{h} to filter an audio recording made elsewhere, \mathbf{x} .
- If you do that, the echoes from the room will be applied to \mathbf{x} and it will sound like \mathbf{x} was recorded in that room.
- This is the basis of **convolution reverb**.

Convolution Reverb in a nutshell

- Record an impulse in the room
- Estimate the impulse response of the room
- Record something in another quiet room (with the microphone up close to the source, so you avoid echoes)
- Apply convolution between your new recording and the impulse response function of the room you want to sound like you recorded in.

Getting $h[n]$: the impulse response

- Walk into a QUIET room with a recorder
- Turn on the recorder
- Clap your hands ONCE (this is your impulse)
- The recording captures the room's impulse response
- ...and other noise (air conditioner, etc.)

Estimating $h[n]$ in a perfect world

- Assume a balloon pop is an impulse. Call it $x[n]$.
- If $x[n]$ is an impulse, then it is a sequence with a 1 at time 0 and a 0 everywhere else.
- Record the balloon pop in a room. Call it $y[n]$.
- If there was no other noise and our recording didn't distort, $y[n]$ is just the convolution of an impulse with $h[n]$.
- This means $y[n] = h[n]$.

Estimating the frequency response H

- Given no noise, we can estimate the impulse response $h[n]$ by estimating the frequency response $H[k]$ in the frequency domain

$$X[k]H[k] = Y[k]$$

$$H[k] = Y[k] / X[k]$$

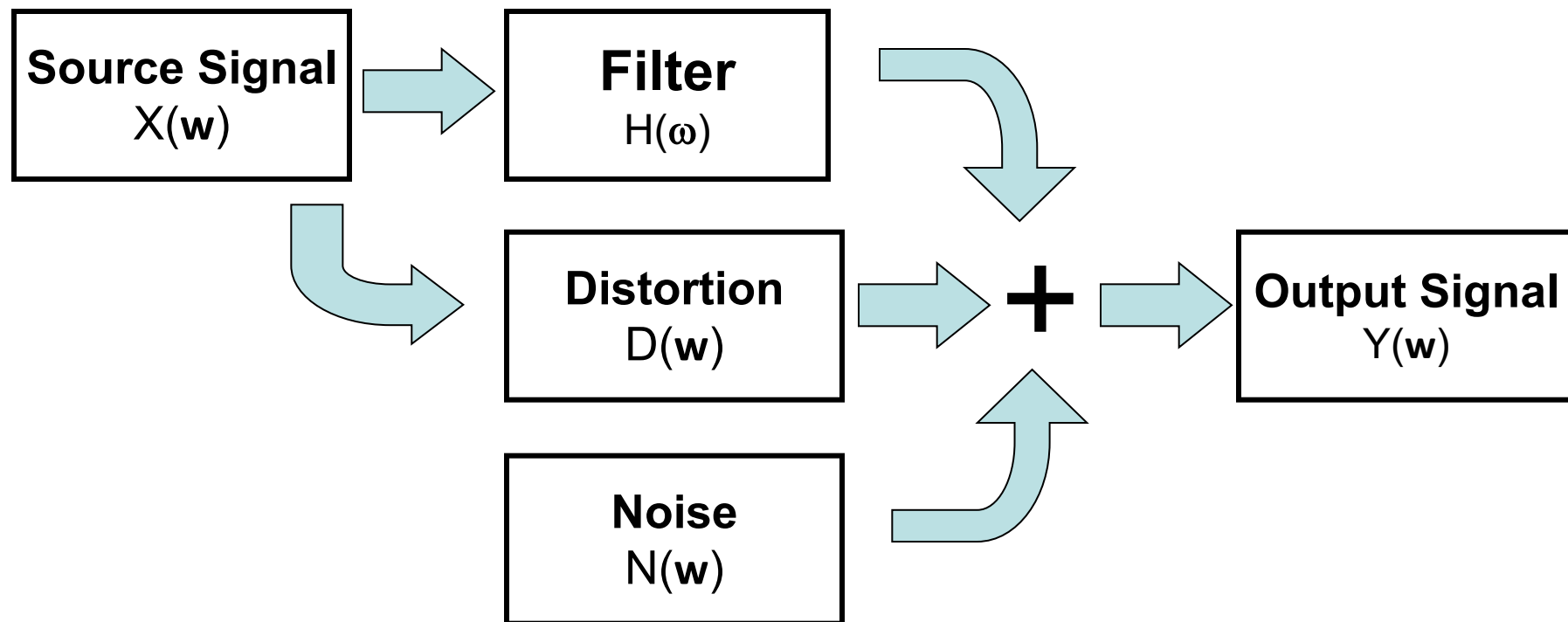
If $x[n]$ is an impulse, then $X[k] = 1$ for all k . Therefore....

$$H[k] = Y[k]$$

Estimating the frequency response H

- But there is noise ...
- What do we do?
- We assume noise is uncorrelated to the signal or the filter
- We assume noise is unbiased (zero mean)
- We try lots of estimates and hope that the noise “washes out” when we average

Noise and Distortion



Distortion and noise make it a lot harder to estimate H

Estimating the frequency response H

- Assume we know what X is (because we made it) and what Y is ('cause we recorded it).
- Hope noise and distortion are not correlated with X .
- Call “noise + distortion” N .
- Then...

$$X[k]H[k] + N[k] = Y[k]$$

Estimating H

- Estimate H a lot of times, in the hopes that the noise will wash out in the mix...

$$\hat{H}[k] \approx \langle Y[k]X[k]^* \rangle / \langle X[k]X[k]^* \rangle$$

where

$$\langle Y[k]X[k]^* \rangle = \frac{1}{N} \sum_{n=1}^N Y_n[k]X_n[k]^*$$

This gives the average value over N experiments. Note the $*$ indicates a complex conjugate.

Caveats

- This only works on the frequencies where there was energy in the input signal X .
- If there wasn't energy at a frequency then we're out of luck.
- So, best to use a broadband sound for X .
- Recall: An impulse is broadband.